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2016
MCM / ICM
Summary Sheet

Summary

We develop models to provide a feasible business plan that a private firm could adopt as a profitable opportunity on space debris removal issues.

We firstly make comprehensive analyses on any possible ambiguity concerning this problem. We especially notice the divergence from different interpretations of *risk*, *benefit*, *cost*, and most importantly, the *time-dependent view* to tackle the problem.

We start with proposing reasonable assumptions and justifying them with enough evidence.

In the Space Debris Distribution model, we firstly approximate the numerical density of space debris with *Least Sequence method* and predict the number of pieces to year 2040 with *Grey Prediction theory*; then we calculate the orbit for a specific piece of space debris. This model lays the foundation for the models ensue.

In the Removal Measure Evaluation model, we firstly develop sub-models to quantify risk, cost and benefit. In order to determine the parameters in these indexes, we combine the *Monte Carlo method* and *Neural Network method* together to make a reasonable approximation. Then we utilize the *Comprehensive Evaluation (CE) method* to quantify the indexes. Considering the demerits of the basic CE method in its failure to describe index fluctuation and the inability to directly provide none-technical consultancy, we develop **three improved editions of the basic model**. We calculate and find that the best measure for debris removal is satellite retrieval.

In the Measure Package model, we regard a measure package as a “new” measure and apply the *Differential Evaluation (DE) algorithm* to calculating its risk, cost and benefit; then we use the Removal Measure model to compare the measure package and the measures adopted solely. We find that a measure package is better than a single measure in term of the three indexes and the **near-optimal measure package** includes 13.14% of space-based laser, 85.02% of land-based laser and 1.84% of satellite retrieval. We then give our interpretations of this result.

We analyze models’ sensitivity against parameter changes; it turns out that influence to results from parameter fluctuation could be neglected.

In order to be proactive to various scenarios, we postulate several conditions and answer “what if” questions based on our models.

We eventually make a review of our work in its strengths and weaknesses; we then submit an Executive Summary to policy makers and medium analysts.

Space Debris Terminator, a Comprehensive Approach

#Team 49423

Contents

1. Introduction

1.1. Problem Description

Neil. Armstrong’s famous words:” that's one small step for man, one giant leap for mankind.” marked the prelude of the space traveling age. From the very moment he stepped onto the moon, it seems nothing could be the obstruction to prevent people from exploring the universe. However, except those mysteries given by nature, some of the remaining obstacles are created by human beings themselves. Among these challenges includes the space debris for mankind to conquer.

Space debris, also named orbital debris, is a collection of man-made objects including abandoned satellites, fragments from collision or erosion and other dumped objects at different orbits around the earth. Circulating around the earth at a velocity as high as nearly 8 kilometers per second, they jeopardize the safety of space traveling.

It is unanimous that space debris could not hinder human beings’ steps to move forward for their better knowledge of nature. At the same time, it is also clear that the removal of space debris requires public participation besides governmental actions. A private firm could address this issue and make it a profitable business; the recent practice of the Space Infrastructure Servicing is one of the examples. We are aiming to develop a model to provide strategies a private firm could adopt for commercial use in space debris location and removal.

Space related issues have intricate technical background; in order to develop a comprehensive model, several ambiguities should be clarified:

- the time dependent view to tackle the problem asks us to fully consider the time variation. However, the specific time span is unclear. Therefore, we will develop models in two different directions, one is to the address the problem in a general time line, the other requires a detailed perspective. One possible example of the former way could be the analyses based on yearly dataset, while the latter one could originate from the data collected through the single mission for debris removal.

- the costs of different removal methods vary from one to another even for the similar operation. The practice of leaser-based destruction facility could best exemplify this point. The land-based laser should be much cheaper than the space-based laser because the latter one requires the construction of the carrying vehicle of the laser destruction facility.

- a private firm benefits from space debris removal mission by being funded by governmental agencies or international organizations. However, the way determines the value of a single debris removal mission is uncertain.

- the possible approaches to remove space debris should be assorted into several different categories. There are two reasons why doing so is necessary:

  a) The risks could be different. For some measures, an abortive mission does not cause more space debris, while some others will cause more debris than before. The number of pieces caused by failed missions could be massive. One example is the destruction of a Chinese satellite in 2007 and the collision between an American satellite and a Russian satellite; their debris now takes up about 1/3 of the space debris family.

  b) A comprehensive plan requires reasonable combination of all possible measures, but the capability of a private firm is limited in the financial and technical aspects. What’s more, different measures bear differences in costs, risks and benefits. Make wise decisions in utilizing limited resources is important, and the first step to make such decisions is to knowing all alternatives in a logical way.

With all uncertainties about the problem settled, now we will make our goals clear:

- conducting a study on the practice of space debris by private firms.

- developing models on approximation and prediction of the distribution of space debris from two types of time-dependent views.

- quantitatively evaluating different types of the debris removal strategies and categorize them into several groups.

- quantitatively analyzing the benefits, costs and risks of certain debris removal strategy.

- testing the model with various cases which contains some extraordinary circumstances to evaluate whether the models could be adopted in different scenarios.

- probing whether or not a business opportunity for a private firm exists according to our work done before.

- depending on the outcome of the opportunity analyses

a) If a profitable opportunity exists, we will make a business plan for the private firm. The plan will include the specific strategies adopted to remove space debris.

b) If a profitable opportunity doesn’t exist, we will put forward methods to minimize the possibility of damage done to the spacecrafts by space debris.

- summarizing all our work and write an Executive Summary to the high level policy makers or media analysts.

1.2. Related Work

Since their potential danger, space debris have arisen international concerns. Now there are two international organizations specially handling the space debris-related issues; they are Inter-Agency Space Debris Coordination Committee (IADC) and the Scientific and Technical Subcommittee of the United Nations Committee on the Peaceful Uses of Outer Space (COPOUS). IADC is working on technical side of the issue; it publicizes the yearly dataset on space debris issues. The COPOUS, on the other hand, emphasizes on the inter-governmental cooperation for the space debris-related research and legislation; it publishes documents including the research proceedings on the protection methods from debris.(NASA 2015)

US Space Surveillance Network (SSN) routinely track the space debris with the diameter greater than 10 cm, and space debris with lesser diameter could be located by land-based radars.(NASA 2016)

Methods to remove the space debris are under study. Many methods are brought up, and their fundamentals, costs, efficiency and other important technical details are discussed. A type of robotic arms have been designed and its feasibility have been confirmed; it could be used to capture debris or to de-orbit the debris’ original position(Nishida and Yoshikawa 2008). The conception of land-based or space-based laser were put forward but the laser frequency and total energy focalized remained uncertain for a long period of time; a laser system construction based on the International Coherent Amplification Network (ICAN) platform has also been designed to remove space debris(Soulard et al. 2014). There are also attempts to focalize the sunlight and target at the specific piece of debris for complete destruction.(Soulard et al. 2014)

2. Assumptions and Justification

- The very piece of space debris under discussion is only affected by earth in gravitation sense.

The universality of the universal gravitation means that any two objects in the universe are affected each other, but it is impossible to consider all other objects at the same time; at the same time, space debris mainly concentrates at the geocentric orbits and therefore mainly affected by the earth's gravitational field. Considering the difficulty and the major effect for the space debris, this assumption is fair and just

- The earth could be regarded as a sphere with uniform composition and density.

a) The equator radius of earth is 6378 km while the polar radius is 6356 km, which are close to each other. As the equator radius and polar radius are the largest value and smallest value respectively. Earth could be seen as a sphere

b) According to the assumption we brought up, the whole family of space debris is only affected by earth's gravitational field; therefore, the selected piece of space debris and earth could be regarded as a two-body system. The characteristics of a two-body system determine that earth could be seen as a uniform object.

- The density of the space debris at a specific orbit is uniform.

From the causes creating space debris we could find that, except rare cases (such as the planned destruction of certain satellite by land control center), the creation of space debris is rather random. Taking a collision as an example, the momentum for different pieces of fragments are different, which means the initial velocity and the mass of them vary a great deal from one to another. What's more, the density of space debris at certain orbits is so high that the Kessler Syndrome may occur. These two scenarios will result in a very high uncertainty in space debris' orbit and thereby their uncertainty of the accurate distribution. In order to develop a quantitative model, we have to posit the uniform distribution of the space debris at a single orbit.

3. Terminology

- Geocentric Orbits(Wikipedia 2016a)

Geocentric orbits are a set of orbits including low-earth-orbit (LEO), medium-earth-orbit (MEO), geosynchronous orbit (GEO) and elliptical orbit. The categorization is made according to their altitudes. When people design a satellite, the top priority is to make clear the function of the satellite, and this determines the orbit it is positioned.

- Kessler Syndrome(Wikipedia 2016b)

The Kessler Syndrome is a scenario in which the density of the space debris at the LEO is high enough that the occurrence of one collision will cause a cascade, which means that the effects of the Kessler Syndrome include not only the damage to a specific spacecraft but the generation of even more space debris.

- Two-body System(Wikipedia 2016c)

A two-body system includes two point particles that merely interact with each other. Some common examples of two-body system are satellite orbiting around earth or a planet orbiting around a star.

4. Models and Solutions

4.1. Reviews on the Business Opportunities in Space for Private Firms

Private firms could also participate in the space-related issues compared to the governmental organizations; they could even make more rapid progress in space exploration since their operations are privately funded and thus could not be hindered by governmental funding meager problems. Several private firms have already stepped into the space business and made profit.

Space-X is a private contractor of the NASA. It determines to lower the costs of space transportation and finally colonize on Mars through technological innovation. With several records such as the first private company to launch satellite and recover it, the first private company to send satellites to the GSO and the first private company that made contracts with the NASA to send supplies to the International Space Station, it grew to be the most influential private contractor in space exploration within less than ten years. It could even replace other governmental agencies in international competition for satellite purchase order.

Motivated by the success of Space-X, many private space projects are on the way.

Space Infrastructure Servicing (SIS) is a project funded by a Canadian private firm which specially focuses on the space debris removal and refueling service provided to satellites. Its strategy is to push the selected piece of space debris to the graveyard orbit.

From the two examples we discussed we could see that the possibility for a private firm to address the space debris is exists and could be a good business opportunity for profits.

We will discuss the details of the space debris removal for a private firm in the models we will develop.

4.1. Model Overview

We will develop models to describe the process illustrated in the **Figure 1.**

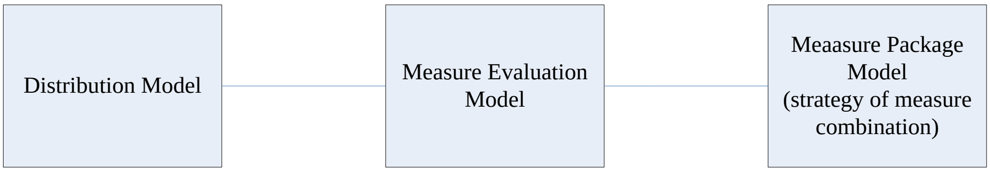

Figure 1. Model Overview

According to goals we made clear before, we will separate the model set into three parts: the first part is about the approximation and prediction about the space debris family over time; the second part includes the assessment of different measures in space debris removal; the last part mainly talks about the strategy of combining different measures for better outcome.

4.2. Space Debris Distribution Model

Analyses

As the distribution of space debris at different altitudes is not uniform and the number of them is largely influenced by human activities, we need a general picture of their distribution before we could track the specific piece of space debris. This general picture should include the approximation of the variation in number and numerical density in the past and the prediction of these two indicators in the future.

For the model in a long time span, we consider that the factors determining the two indicators could be intricate in their technical background and be massive in number. It is believed that the grey prediction method boasts its advantage of the single dependence on the dataset itself; in order to make good use of the data collected from the IADC, we apply grey prediction method to the evaluation of the two indicators.

For the model in a short time span, we could locate the position of a single piece of space debris with the help of infrared sensor telescope since it is a space-based facility that could penetrate potential obstruction in the universe and detect the specific object circuiting around the earth, including space debris of certain size. In order to meet the accuracy criterion, we could do the same observation for many times. We then could apply numerical methods to the collected dataset and find the approximate trajectory of the selected piece of space debris.(Toyoda et al. 2009)

Models and Solutions

• Distribution Model in Long Time Span

We collected the dataset provided by the IADC. The dataset contains the number of total objects, the number of fragmentation debris around the earth and other three indexes since 1961, the year people first sent an astronaut to the universe. The fragmentation debris means the total number of debris caused by manned activities such as active destruction or unmanned activities including erosion and collisions within the debris family.

Let $X^{(0)}$ be the series containing the number of fragmentation debris

$$X^{(0)} = \left(x_{(1)}^{(0)}, x_{(2)}^{(0)}, \cdots, x_{(k)}^{(0)}\right)$$

$x_{(k)}^{(0)}$ is the number of fragmentation debris of the selected year.

The condition to use the grey prediction method is the ratio $\lambda(k) = {x_{(k-1)}^{(0)}} \big/ {x_{(k)}^{(0)}}$ lies within the interval $\left[e^{-\frac{2}{n+2}}, e^{\frac{2}{n+2}} \right]$, where *n* is the size of the given dataset. We calculate the ratio and find that all of them lie within this interval, so the condition is met.

The grey prediction method requires *grey generalization* at the beginning. The grey generalization is a transform of the given series; it has two different outputs $X^{(1)}$ and $Z^{(1)}$ which respectively represents the old dataset's characteristics, they are described as follows:

$$\begin{aligned} X^{(1)} &= \left(x_{(1)}^{(1)}, x_{(2)}^{(1)}, \cdots, x_{(k)}^{(1)}\right) \\ &= \left(x_{(1)}^{(0)}, x_{(1)}^{(0)} + x_{(2)}^{(0)}, \cdots, \sum_{i=1}^{k} x_{(i)}^{(0)} \right) \\ Z^{(1)} &= \left(z_{(1)}^{(1)}, z_{(2)}^{(1)}, \cdots, z_{(k)}^{(1)}\right) \\ &= \left(\frac{1}{2}(x_{(1)}^{(1)} + x_{(2)}^{(1)}), \frac{1}{2}(x_{(2)}^{(1)} + x_{(3)}^{(1)}), \cdots, \frac{1}{2}(x_{(k-1)}^{(1)} + x_{(k)}^{(1)})\right) \end{aligned}$$

The two transformed series $X^{(1)}$ and $Z^{(1)}$ will then be used to establish simultaneous differential equations; this equation set and is described as:

$$\begin{aligned} & X'^{(0)} + aZ^{(0)} = b \\ & \frac{dX^{(1)}}{dt} + aX^{(1)} = b \end{aligned} \tag{4.1}$$

The vector $X'^{(0)}$ is special because we exclude the element $x_{(1)}^{(0)}$ from it according to the grey prediction theory; it is now congruent in dimension with the vector $Z^{(0)}$.

We solve this equation set and we get

$$x_{(k+1)}^{(1)} = \left(x_{(1)}^{(0)} - \frac{b}{a} \right) e^{-ak} + \frac{b}{a}$$

The parameters a and b are remains to be settled.

The grey prediction theory claims that the optimal parameters of the are those minimize the function $J\left(\hat{u}\right)$, which is described as:

$$J\left(\hat{u}\right)=(Y-B\hat{u})^T(Y-B\hat{u})$$

where $\hat{u}$ is $\left(a,b\right)^T$, Y is $\left(x_{(2)}^{(0)},x_{(3)}^{(0)},\cdots,x_{(k)}^{(0)}\right)^T$ and B is $\begin{pmatrix}-z_{(2)}^{(1)} & 1\\ -z_{(3)}^{(1)} & 1\\ \vdots & \vdots\\ -z_{(k)}^{(1)} & 1\end{pmatrix}$.

We apply the numerical methods to minimize $J\left(\hat{u}\right)$, and we get the equations of with variables Y and B

$$\hat{u}=\left(B^TB\right)^{-1}B^TY$$

The values of *a* and *b* are-0.0795 and 1252.1 respectively.

Therefore, the solutions to the equation (4.1) is

$$x_{(k+1)}^{(1)}=\left(x_{(1)}^{(0)}+15749.68\right)e^{0.0795k}-15749.68$$

Use the transformed series, we could find the value in any year, **Figure 2** displays the approximation of the numerical density and number in the past and prediction of number in the near future.

• Distribution Model in Short Time Span

We assumed that the earth itself is a sphere with even density and composition and it lies at one of the focal points of the selected piece of space debris' orbit. As part of the *two-body system*, earth mainly determines the selected piece's moment. The assumption we brought up allows us to directly apply Newton's law of gravitation to describing the selected piece's moment without being disturbed by the earth's characteristics of uneven density and irregular shape.

The Newton's law of gravitation is described as:

$$\frac{d^2r}{dt^2}=\frac{GM}{r^2}$$

where is the gravitational constant, is the mass of the earth and is the radius from the orbit of the selected piece of debris to earth's center of mass.

The radius we put forward has a quantitative relation with the length of the semi-major axis denoted by *a*, and the eccentricity denoted by *e*

$$r = \frac{axis(1-e^2)}{1+e\cos\theta}$$

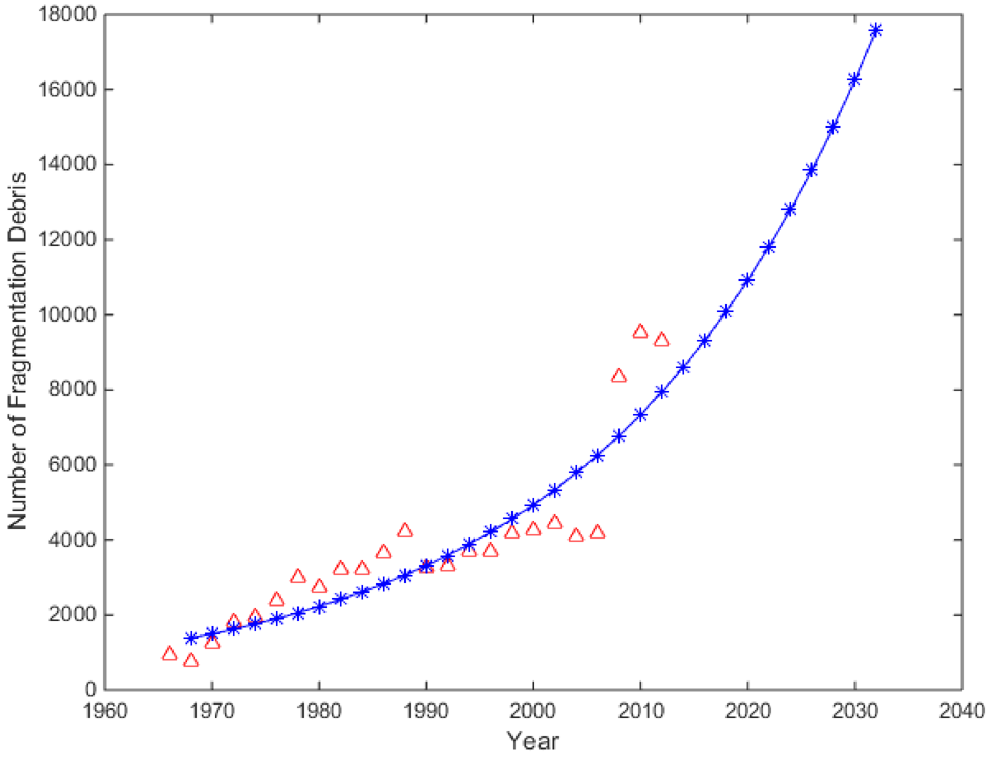

Figure 2. The Prediction of Fragmentation Debris to 2040

We also conduct the similar numerical methods as in the distribution model. We could acquire the value of *r*

$$\hat{r} = \frac{axis(1-\hat{e}^2)}{1+\hat{e}\cos\theta}$$

- ## Approximation of Numerical Density

From the dataset collected from the IADC, we acquire the datum of the fragmentation number and the object number by year 2011. In this dataset, we find that the relations between these two values are seemingly proportional. We then apply the linear regression method to these two datasets to test whether the linearity exist or not.

According to **Figure 3** we now could assert that the relation between these two values is proportional. Therefore, we could use the distribution of all objects to replace the distribution of fragmentation debris.

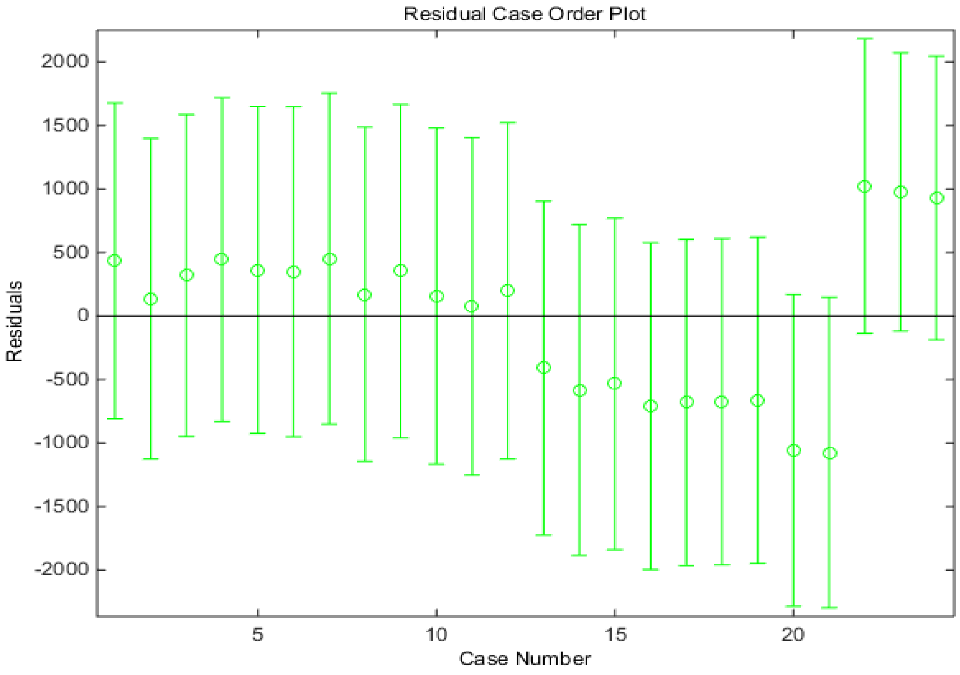

Figure 3. Residual Case Plot

At the same time, we also collected the datum of operational satellites at different orbits, therefore, we could approximate the numerical density of fragmentation debris, just as it is depicted in **Figure 4.**

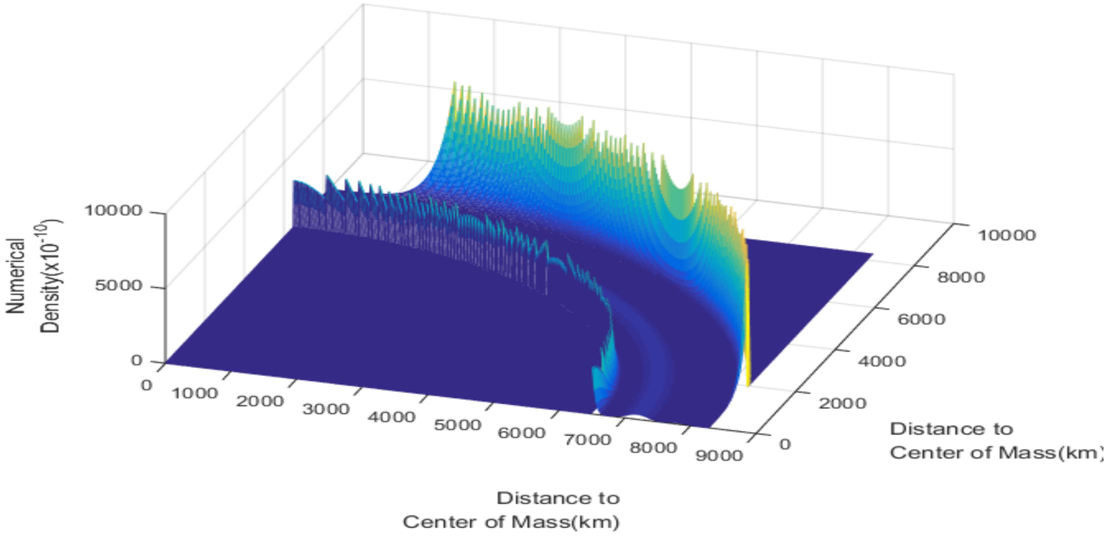

Figure 4. Numerical Density over the Distance to the Center of Mass

Sensitivity Analyses

In order to analyze the sensitivity of the model to the parameters *a* and *e*, we intentionally add noise to the calculated value of *r*; the additive noise obeys certain normal distribution, we let its standard deviation be 1 and its mean value be 0, that is

$$noise \sim N\left(0,1\right)$$

From the **Figure 5** we could see that the output value of remains constant near one specific orbit, we could also claim that our model is robust against parameter fluctuation.

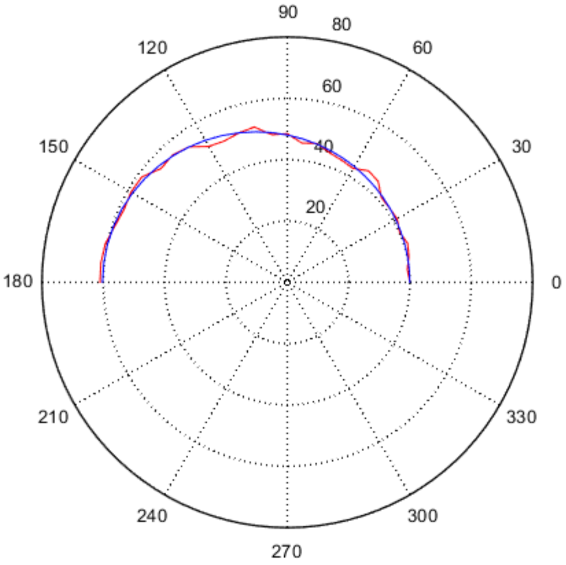

Figure 5. The Orbits before and after Additive Manned Noise

4.2.Removal Measure Evaluation Model

Analyses

- ### Classifications of Different Types of Space Debris

The space debris should be categorized into several groups to evaluate their dangers posed on the space traveling. The criteria for this categorization could be size, mass, numerical density and other related factors.

According to the difficulty to track them, the space debris family could be categorized into three groups.

Table 1

The Classifications of Different Types of Space Debris

Category	Diameter	Number	Measures for Tracking
1	Larger than 10cm	23,000	Quantitative through special surveillance network
2	1~10cm	600,000	Quantitative through land-based radar
3	0.1~1cm	70 to 80 million	Qualitative
4	Smaller than 0.1cm	10^{13} to 10^{14}	Impossible to track

Here we define the debris in category 1 ”big piece”, the debris in category 2 “middle piece”, and those in category 3 the “small piece”.

• Classifications of Different Space Debris Removal Strategies

The strategies for active removal of space debris could be categorized into several groups according to whether there exists direct contact between the removal facility the selected piece of debris. Differentiating from each other in costs, efficiency and other aspects, the measures could be further separated. The measures which have been deeply studied are shown in **Figure 4.**

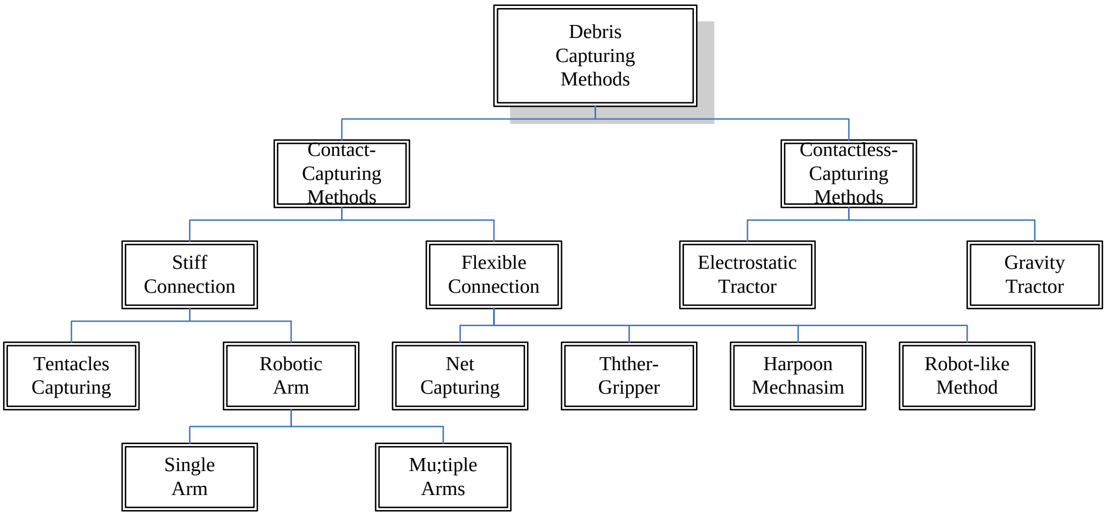

Figure 6. The Measures used for Space Debris Removal(Shan, Guo and Gill 2015)

Considering that the various forms of removal strategies differ from each other greatly, the precise estimation of the specific form of strategy in risks, benefits and costs could be difficult. Thus, we pick three representatives based on the criteria

a) The selected measures should be comparable.

b) The selected measures should bear significant differences for comparison.

Therefore, we choose the three different measures; they are land-based laser, space-based laser and special satellite launched for retrieving.

• Overview of the Comprehensive Evaluation Model

Comprehensive Evaluation method provides a possible way to analyze the problem with considerable ambiguities since many indexes in an evaluation problem could not be quantified directly. As the risks, costs and benefits of a removal measure could not be acquired through certain dataset, the application of Comprehensive Evaluation method is reasonable.

Models and Solutions

• Process of the Measure Evaluation Model

The process of Measure Evaluation model is shown is **Figure 5.**

• Preparation for the Comprehensive Evaluation Model

The risks, costs and benefits of removal measures should be quantified since the specific values are essential in the model. However, we could not directly get the dataset of three indexes.

The values of risks, costs and benefits could be formulated by the models to be developed later. As the numerical density changes with the change of variation of space debris in number, we will first develop the numerical density model.

1. Numerical Density Model

The total number of space debris at a certain orbit varies in number over the removal process; therefore, the calculation of numerical density requires iteration.

The numerical density approximated in the Distribution Model is the density of the traceable pieces of space debris; to estimate the numerical density of the whole space debris family, we calculate the proportion of each type of debris in the space debris family and denote them with p_{small}, p_{middle} and p_{big} respectively.

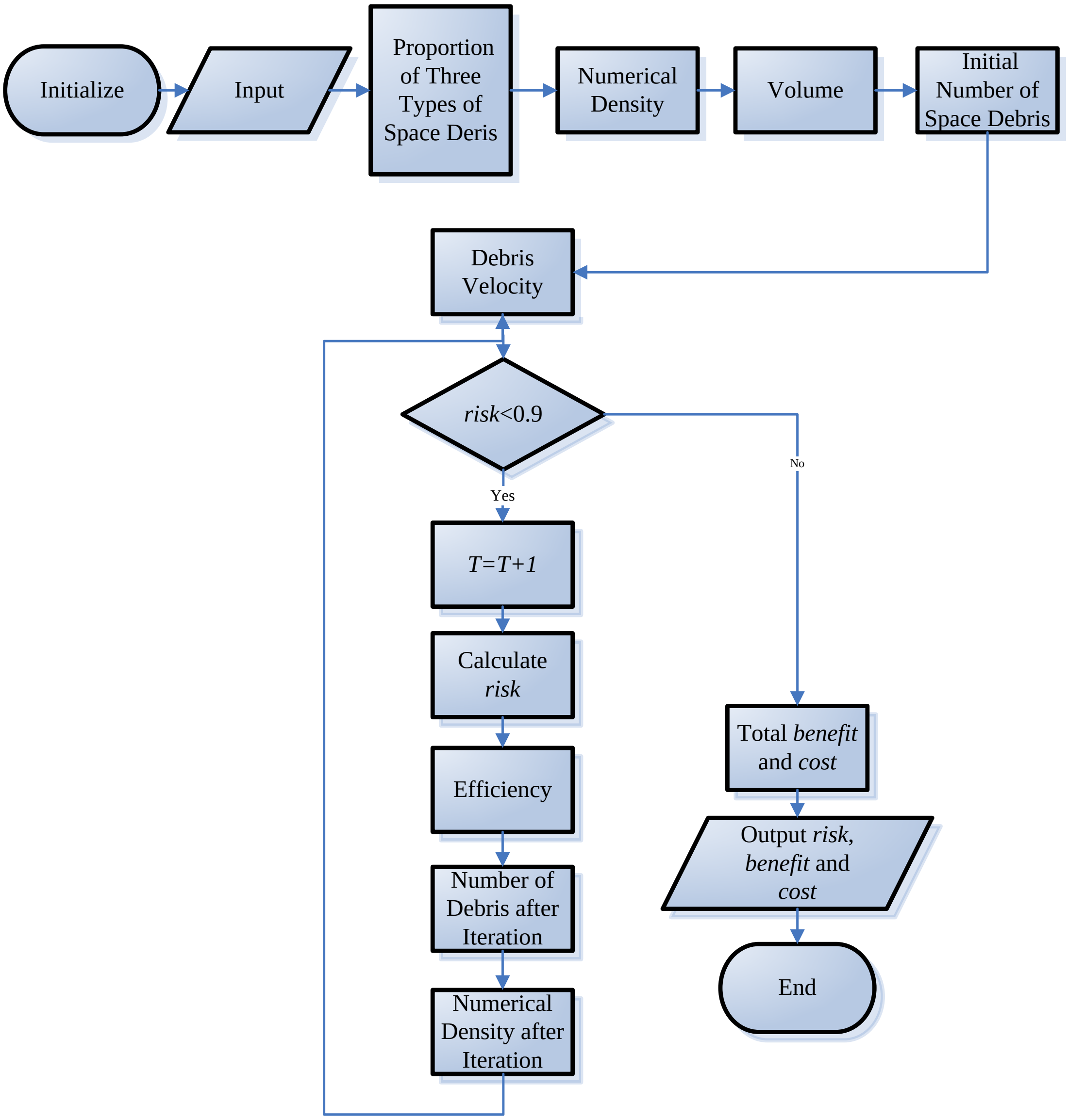

Figure 7. The Flow Chart of Measure Evaluation Model

The total volume for debris family to move at a single orbit is $V = 2\pi rs$, where r is the orbital radius and s is the space reserved for space-based facilities to function.

After several removal missions, the debris remains is

$$n_{remaining}\left(T\right) = n_{remaining}\left(T-1\right) - \eta\left(T\right)$$

where $T = 1, 2, \cdots$ .Specially, we define $n_{initial} = n_{remaining}(0)$; it is obvious that $n_{initial} = \frac{\rho_{initial}}{p_{big}} \frac{n_{debris}}{n_{object}} V$, where n_{debris} and n_{object} have been calculated in the Distribution Model.

Therefor**e,** the numerical density at moment T is $\rho(T) = n_{remaining}(T) \big/ p_{big}V$.

With a description of numerical density over time, we could derive the number of debris removed during a single T period, which could also be comprehended as efficiency. It is described as:

$$\eta(T) = \frac{\rho(T)}{p_{big}} v_{remove}\left(p_{small} + (1-k_1)\,p_{middle} + (1-k_2)\,p_{big}\right)$$

Note that $(1-k_1)\,p_{middle}$ means that the removal of middle pieces could generate more small pieces and cause the diminution in middle pieces. Similarly, $(1-k_2)\,p_{big}$ means that the removal of big pieces could generate more middle pieces and cause the diminution in big pieces. Therefore, $p_{small} + (1-k_1)\,p_{middle} + (1-k_2)\,p_{big}$ could describe the debris family after one removal mission.

2. Risk Model

For a certain piece of space debris, the velocity it travels around the earth could be derived with the help of Newton's law of gravitation

$$v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{r}\right)}$$

where the product GM is a constant called geocentric gravitational constant; its value is 3.986×10^5 km/s^2.

Assuming that the satellite to be positioned for removal mission is a cylinder with the sectional area *S*. The average times of collision over one period *T* is

$$N_c = \rho(T)SvT$$

The probability for times of collisions obey the Poisson distribution, that is $P(i) = \frac{N_c{}^i}{i!}e^{-i}$ .We could then calculate the probability of zero collisions is $P(0) = e^{-N_c}$ .Therefore, the probability for collisions is

$$P_c = 1 - e^{-N_c}$$

The calculation of *risk* is based on P_c; *risk* should have an upper bound for it could not increase without limit. Inspired by the Logistic model for population increase description, *risk* could be calculated as follows:

$$risk = \begin{cases} \left[\dfrac{2\left(P_c - 0.05\right)}{0.9} \right]^2, 0.05 < P_c \le 0.5 \\ 1 - \left[\dfrac{2\left(P_c - 0.05\right)}{0.9} \right]^2, 0.5 < P_c < 1 \end{cases}$$

Note that the risk is be is different when discussing land-based laser as it is much safer to carry out removal mission on earth. Considering the possibility that the land-based laser hit the operational satellite, we define the risk of land-based laser

$$risk = \frac{m_{satellite}}{m_{debris}}$$

3. Benefit Model

The discussion of benefit should include two situations; the first is the success of a removal mission while the second is a failure.

Before the detailed discussion of *benefit*, we should first define these two situations.

As the removal of a single piece of space debris makes little difference to the increase of space travelling safety, a single debris removal mission could only be defined successful when the orbit being cleaned could hardly pose any threat onto any space exploration mission. Considering the distribution of the space debris and the protection tactics used by the ordinary spacecraft, such scenario could only be accomplished when most of the traceable space debris, including three types, is removed through any possible measures.

It is clear that the risk directly determines whether a mission could be successful or not, we let the *risk*=0.9 as the border of a success and a failure. The detailed expression of benefit is as follows:

$$benefit = \begin{cases} V_{orbit} + \left[n_{initial} - n_{remaining}\left(T\right) \right] V_{removal}, risk < 0.9 \\ \left[n_{initial} - n_{remaining}\left(T\right) \right] V_{removal}, risk \ge 0.9 \end{cases}$$

where V_{orbit} is the orbital benefits created by a successful mission, while $V_{removal}$ is the benefits created by the risks lowered by debris removal.

4. Cost Model

The discussion of cost should also include two situations as the Benefit Model since a failure could cause a satellite loss while a success will not except some costs in repairing or maintenance. However, both of the two situations have costs in construction, which we define as fixed costs; it is denoted by C_{fixed} .

The detailed expression of costs is as follows:

$$cost = \begin{cases} C_{fixed} + C_{use}T + risk \cdot C_{fixed}, risk < 0.9 \\ C_{fixed} + C_{use}T + C_{fixed}, risk \geq 0.9 \end{cases}$$

where C_{use} is the costs in using the removal facility.

These three models are sound but incomplete for their parameters remain unknown up to now. The unknown parameters include

The parameters of this model set could not yield without certain data generation process; in order to best approximate the unknown parameters, we combine the Monte Carlo method and Neutral Network method together.

With a complete model set to quantify risks, costs and benefits we now could acquire a dataset of the all three indexes.

One dataset acquired through the methods we talked about is shown in **Table 2.**

Table 2.

One Example Dataset

	Benefit	Risk	Cost
Space-based laser	1130.2521306232	0.107117193440142	933.135158032043
Land-based laser	1118.32161955308	0.000100000000000	1402.50000000000
Satellite retrieval	119.099101486537	0.901419110855645	1103.60000000000

- ## Basic Comprehensive Evaluation Model

Since the indexes *cost, risk* and *benefit* are not consistent in their units and typical range of distribution, we first normalize the indexes:

$$
\begin{aligned}
benefit_j &= \frac{b_j - b_{\min}}{b_{\max} - b_{\min}} \\
cost_j &= \frac{c_{\max} - c_j}{c_{\max} - c_{\min}} \\
risk_j &= \frac{r_{\max} - r_j}{r_{\max} - c_{\min}}
\end{aligned}
\tag{4.2}
$$

where j=1,2,3. Note that the indexes are different, for *cost* and *risk*, their values will be “better” when they are larger while the value of *benefit* will be “better” when they are smaller, this could reflect in the calculation we did in the equation (4.2).

Now we acquire the *evaluation matrix*

$$
W = \begin{pmatrix} 1 & 0.93 & 1 \\ 0.99 & 1 & 0 \\ 0 & 0 & 0.64 \end{pmatrix}
$$

where the lines represent *benefit*, *risk* and *cost* respectively and columns represent three different measures(land-based laser, space-based laser and satellite retrieval) we chose to remove space debris,

The indexes play different roles in a measure’s evaluation, which is embodied in their importance in single evaluation. We apply Entropy Evaluation method to determine the weights of *risk*, *benefit* and *cost* under different conditions. The process of Entropy Evaluation method is illustrated in the **Figure 8**.

$$
\boxed{I_j = -\frac{1}{\ln(3)} \sum_{i=1}^{3} \left(\frac{x_{ij}}{\sum_{i=1}^{3} x_{ij}} \right) \cdot \ln \left(\frac{x_{ij}}{\sum_{i=1}^{3} x_{ij}} \right),\ j = 1,2,3} \rightarrow \boxed{r_j = 1 - I_j,\ j = 1,2,3} \rightarrow \boxed{w_j = \frac{r_j}{\sum_{j=1}^{3} r_j},\ j = 1,2,3}
$$

Figure 8. The Process of the Entropy Evaluation Method

Therefore, the evaluation matrix is

$$
W' = \left(w_1, w_2, w_3 \right) \cdot W
$$

Now we could quantitatively evaluate three measures with the following equation:

$$
y_i = \sum_{j=1}^{3} w_j \cdot x_{ij}
$$

The results of the Basic Comprehensive method is displayed in the **Table**

Table 3

The Results of the Basic Comprehensive Method

Measure	y
Space-based Laser	0.9773
Land-based Laser	0.6497
Satellite Retrieval	0.2206

• Extension of the Basic Comprehensive Evaluation Model

The Basic Comprehensive Model leaves out some important factors, which are

a) the potential changes in the indexes.

b) the differences in removal efficiency for same measure at different altitudes

c) a more direct reference for policy makers who don't have technical background

d) taking GEO, which is a single precious orbit for satellites, into account.

For the problem a), the values of *cost* and *benefit* may fluctuate as a result of the advances in technology and the governmental policy support. Therefore, we use the parameter k_b and k_c to modify them.

We are inspired by Logistic Model used to describe population fluctuation. Considering the governmental support and technological advances both have upper bounds. We calculate the two parameters in the following approach:

$$k_b = 1 + \frac{1}{1 + e^{-\frac{n_{remaining}(T) - 15000}{4000}}}$$

$$k_c = 1 - \frac{0.5}{1 + e^{-\frac{year - 2010}{10}}}$$

Therefore, the modified evaluation matrix is

$$W'' = \begin{pmatrix} k_b \\ 1 \\ k_c \end{pmatrix} \cdot W'$$

Now we could calculate the modified evaluation of different removal measures.

For the problem b), we use the Basic Comprehensive Model at a given altitude; we choose 16 orbits in total from 300 km to 1800 km with an increment 100 km.

For the problem c), we rank the calculated evaluation values in descending order. According to the rank, we let the score of different measures at a specific altitude 3, 2 and 1. The sum of the scores at different altitudes for one measure is a overall evaluation of the measure. The results are shown in the **Table 4.**

Table 4

The Results of Measure Evaluation considering Altitudes

Altitude(km)	Space-based Laser	Land-based Laser	Satellite Retrieval
300	2	1	3
400	2	1	3
500	2	1	3
600	1	3	2
700	1	3	2
800	1	3	2
900	1	3	2
1000	1	3	2
1100	2	1	3
1200	2	1	3
1300	2	1	3
1400	3	2	1
1500	3	2	1
1600	2	1	3
1700	2	1	3
1800	2	1	3
Total Score	29	28	39

Table 4 could be exhibited in a more sensible way just as **Figure 9.**

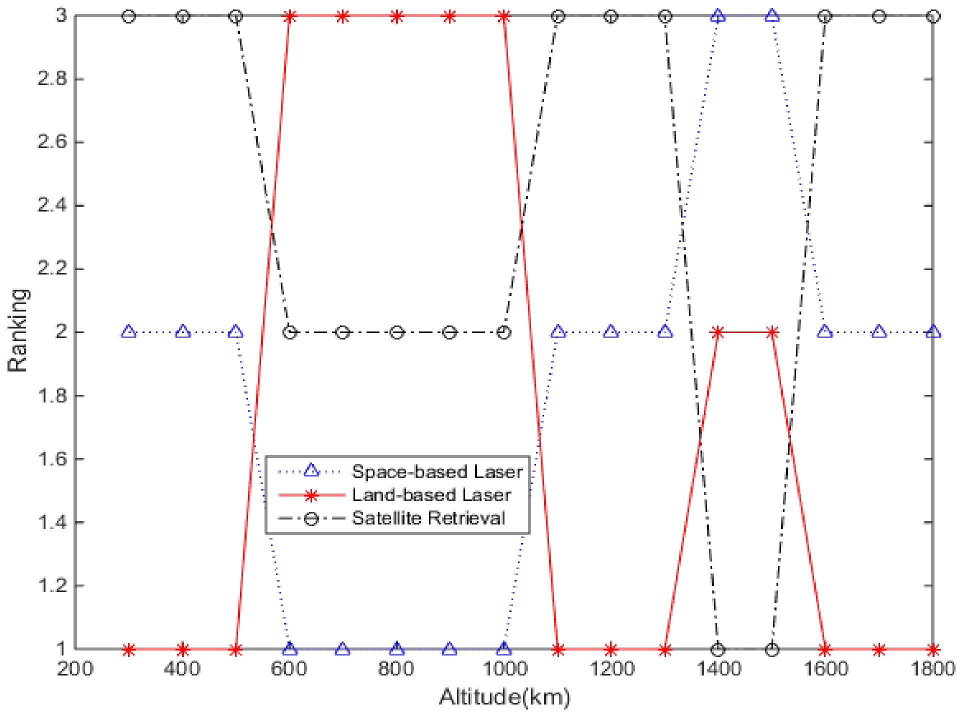

Figure 9. Ranking of Different Measures at Different Altitudes

Now we could provide a perspicuous interpretation of our results for further use.

For problem d), as the land-based laser does have the ability to reach the target at the GEO, only two methods could be applied.

We calculate the *benefit*, *cost*, *risk* of space-based laser and satellite retrieval. The results are shown in the **Table 5.**

Table 5.

Evaluation for GEO Debris Removal Strategy

	benefit	*risk*	*cost*
Space-based Laser	22.47	0.90	1715
Satellite Retrieval	1044.5	0.13	608.5

We could find that satellite retrieval method is better and should be adopted in GEO debris removal

Sensitivity Analyses

The risk, cost and benefit sub-models include various parameters that may fluctuate and therefore influence the output. We will conduct sensitivity analyses of the Measure Evaluation model through manned deviation in parameters.

The **Figure 7** shows the changes in output after adding deviation of different percentages.

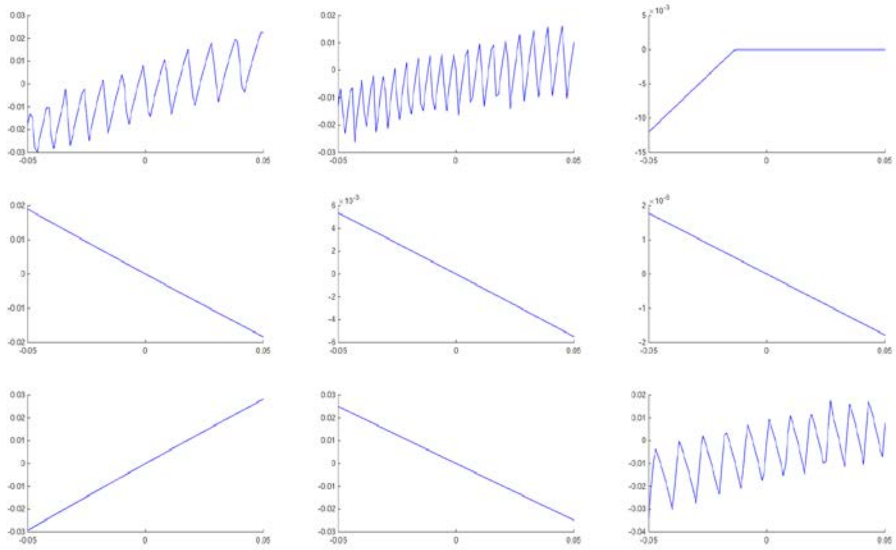

Figure 10. The Output Fluctuation(%) Derived from Additive Deviation(%)

As the output changes from the changes all of the nine parameters in this model are less than that of additive deviation, we could say that our models are not sensitive to parameter changes.

4.3.Measure Package Model

Analyses

The adoption of a single measure might not acquire the best outcome in space debris removal considering the characteristics of these measures. Therefore, the measures we chose (including land-based laser, space-based laser and satellite retrieval) could be combined to achieve a better outcome.

A good measure package should lower the costs and risks, but increase the benefits. At the same time, the alternative of a measure package should also be compared with the three single measures. Therefore, the selection of a feasible measure package could be done within two steps, which are

a) making a good balance between three different measures and put forward a plan;

b) comparing the measure package with the measures adopted solely.

In order to make good uses of the models having been built, we apply the Comprehensive Evaluation Model to the package evaluation. We regard the measure package as a “new” measure and the comparison is made in this way.

Models and Solutions

• Differential Evolutionary Algorithm

In order to select the best measure package with larger benefits and lower risks, costs simultaneously, we use the Differential Evolutionary (DE) algorithm. DE is a algorithm based on the simulation of creatures’ evolution; it could find an individual topping the group in its “competitive power” after numerous rounds of transform including mutation, crossover and selection.(Pavone, Narzisi and Nicosia 2012)

Let the size of group $NP=100$, then we randomly generate vectors with the elements representing the proportion of space-based laser, land-based laser and satellite retrieval to be adopted in a debris removal mission respectively; these proportions obey the uniform distribution.

To be more specific, the vector is $x_i^{(0)}$ while the group is as follows accordingly:

$$X^{(0)} = \left(x_1^{(0)}, x_2^{(0)}, \cdots, x_{NP}^{(0)} \right)$$

In the mutation process, the group generates another group $M^{(t+1)}$ consisting of mutated individuals $m_i^{(t+1)}$ according to the following equation:

$$m_i^{(t+1)} = x_{r_1}^{(t)} + k \left(x_{r_2}^{(t)} - x_{r_3}^{(t)} \right)$$

where $k = F_0 \cdot 2^{\lambda}$ and $\lambda = \exp(1 - \frac{Iter_{\max}}{1 + Iter_{\max} - Iter_{present}})$

We call this new group mutated group.

In the crossover process, mutant operand CR is defined as

$$CR = CR_{\min} + \frac{Iter_{present}}{Iter_{\max}} (CR_{\max} - CR_{\min})$$

where $CR_{\max}$ and $CR_{\max}$ have to be assigned by users.

For the *j*th element belonging to the vector $x_i^{(t)}$ of the group, which is denoted by $x_{i,j}^{(t)}$, it could be replaced by the reciprocal element in the mutated group at certain probability, that is

$$u_{i,j}^{(t+1)} = \begin{cases} m_{i,j}^{(t)}, rand(0,1) \le CR \\ x_{i,j}^{(t)}, rand(0,1) > CR \end{cases}$$

where $rand\left(0,1\right)$ means the random number between interval $\left[0,1\right]$ obeying uniform distribution.

In the selection process, we make use of the sub-models in Measure Evaluation model, including risk model, cost model and benefit model, to find a better vector between $x_i^{(t)}$ and $m_i^{(t)}$

After $Iter_{\max}$ rounds of iterations, this DE based model could yield a near-optimal vector.

• Comparison between the Single Measure and Measure Package

We use Comprehensive Evaluation method to compare the measure package and single measures. To be more specific, we regard the measure package as a “new” measure and then conduct the exactly same calculation as that in Comprehensive Evaluation method.

The results are shown in **Table 6**

Table 6

The Results of the Near-Optimal Measure Package

Altitude(km)	Space-based laser	Land-based laser	Satellite retrieval	Measure Package
300	2	1	3	4
400	2	1	3	4
500	2	1	3	4
600	1	3	2	4
700	1	3	2	4
800	1	3	2	4
900	1	3	2	4
1000	1	3	2	4
1100	2	1	3	4

1200	2	1	3	4
1300	2	1	3	4
1400	4	2	1	3
1500	4	2	1	3
1600	2	1	3	4
1700	2	1	3	4
1800	2	1	3	4
Total Score	31	28	39	62

Table 5 could be exhibited in a more sensible way just as **Figure 11.**

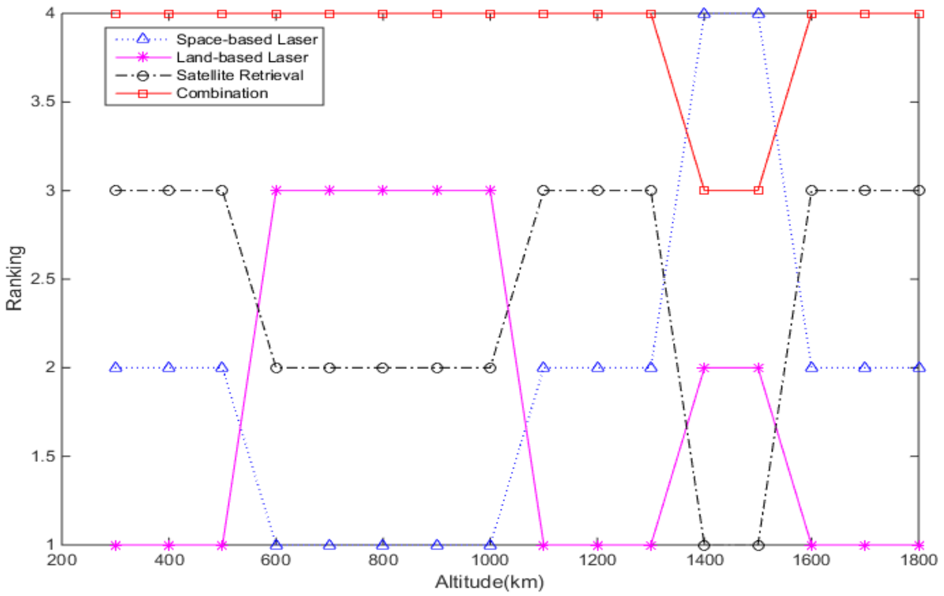

Figure 11. Ranking of Different Measures at Different Altitudes (including measure package)

- **Interpretation of the Result**

From the best measure package acquired, we could see that the land-based laser has a predominate proportion, which seems to contradict the results in the Measure Evaluation model, where land-based laser ranks last among the three measures.

This result could be interpreted through the **Table 4.** The reason why land-based laser has great proportion in a measure package is land-based laser does well in high density area, where risks will arise abruptly and the costs of space-based methods, including satellite retrieval and space-based laser, will increase greatly; at the same time, the cleaning of orbits of debris of high concentration will make considerable benefits.

Sensitivity Analyses

As a heuristic algorithm, Differential Evolutionary Algorithm does not have generally accepted metrics in choosing parameters, and therefore does not necessitates sensitivity analyses since such analyses could not describe model’s resilience against parameter fluctuation.

4.4.Answers to “What if” Questions

The space-related issues are complicated since the number of factors influencing certain parameter in our models could be massive. We will talk about three different situations to show that our models are adaptive to different circumstances

a) Influences from natural factors. Effects such as solar maximum, a period of time sun has vigorous moments, could cause further decomposition of space debris. We considered this cascade effect in sub-model of numerical density. As our models for evaluation are all based on this numerical density model, we could say that our models are resilient to such natural factors.

b) Influences from human-related factors. We considered the demerits of the Basic Comprehensive Evaluation model and put human-related factors, including technological advances and governmental policy support, into account and finally make a improved edition of basic model. We could say our models are comprehensive for human-related factors.

4.5. Feasibility of Debris Removal as a Business

Considering the net benefit which is calculated with $benefit_{net} = benefit - cost$

We acquire the diagrams describing the measures used solely and the measure package. The results are shown in **Figure 12.**

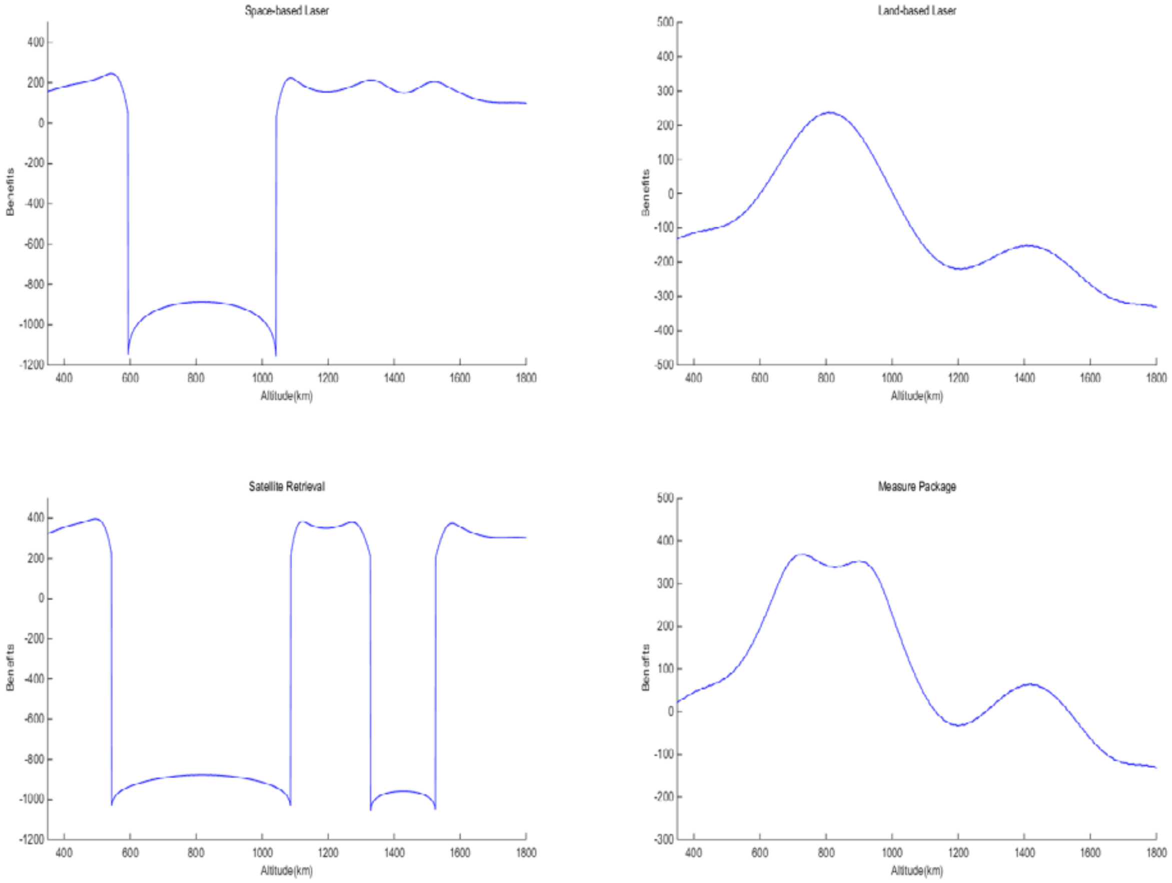

Figure 12. Net Benefit of Different Measures(including measure package)

From the Figure we could see that a measure package could gain profits at most of the altitudes, which is better than the measures used solely. Therefore, measure package could be a business opportunity for a private firm to seize.

5. Conclusion

a) Of all the removal measures we choose, the best measure should be adopted solely is satellite retrieval.

b) Of all different ways in combination of different measures, one near-optimal way has proportions of land-based laser, space-based laser and satellite retrieval 85.02%, 13.14% and 1.84% respectively.

c) The near-optimal measure package could be a feasible business opportunity for a private firm.

6. Strengths and Weaknesses Analyses

7.1. Strengths

a) We have comprehensive analyses of the problem; as our models are based on these analyses, our models could have a relatively more precise description of the problem.

b) The outputs of our models are not largely influenced by parameter fluctuation according to our sensitivity analyses

c) We apply many different methods in developing medals; therefore, our models bear merits of different methods and demerits are overcome.

d) We have extensions of model in Measure Evaluation model. We find problem of Basic Comprehensive Model and develop three improved editions.

7.2. Weaknesses

a) We apply entropy evaluation method in determining the weight values in the Comprehensive Evaluation model; however, there are many different ways to calculate weights. As weights determine the accuracy of evaluation, we could not assert that the entropy method could help us accomplish best accuracy.

b) The extent between optimal and near optimal values in the Differential Evolutionary algorithm depends on $CR, CR_{\min}$ and $CR_{\max}$, which are all subjectively chosen by the user. However, there does not exist a unanimous method to choose them, therefore, we could not guarantee our choice could help the model yield a better outcome than other choices.

8. Executive Report

Nowadays, an increasing number of scientists agree that space debris is a serious problem for development of space exploration enterprise. Space debris includes old satellites, spent abandoned upper stages and fragments from collisions and erosions. What is even worse is the fact that those collisions and erosions can be caused by debris itself. According to statistics, the mass of space debris is expanding over 3 kilograms till 2013 and there is no sign of slowing down.

To address the space debris problem, a number of methods have been proposed. Basically, there are three ways to remove the space debris. First, high energy lasers target specific pieces of debris. Second, large satellites are designed to sweep up the debris. Third, manipulators are able to capture debris. However, the explanative of these methods needs more proof in practice.

Our team has developed models on approximation and prediction of the distribution of space debris from two types of time-dependent views. We conclude that a profitable opportunity exists and we made a plan.

Table. The Assessment of Different Measures at Different Altitudes

	300-500km	600-1000km	1100-1300km	1400-1500km	1600-1800km	Total Score
Space-based laser	6	5	6	6	6	29
Land-based laser	3	15	3	4	3	28
Satellite retrieval	9	10	9	2	9	39

First of all, we quantitatively evaluate different types of the debris removal strategies and categorize them into three groups which are land-based lasers removal, space-based lasers removal and satellite retrieval.

We score the three strategies from the benefits, costs and risks of certain debris removal strategy which are based on low earth orbit heights.(See **Table**)

In conclusion, the satellite retrieval is the highest rated method and is the best alternative and this should be further developed.

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